**Exponential Distribution**

Home Work

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**Statement of Problem:**

For this problem we were asked to model the remission time of 21 patients with acute Leukemia us the exponential distribution. The data for the 21 patients can be seen in Table 1.

1. Construct a linear relationship between the remission time and the cumulative distribution function. Estimate from the graph.
2. Use Max Likelihood to estimate Use the Likelihood ration to determine the region of support. Also, approximate -2log using a quadratic and determine the approximate 90% confidence interval.

**Problem Solution:**

**Part A)**

In this section a linear relationship between the cumulative distribution function and the remission time was developed. If the exponential distribution is a good estimation for the data a line should fit the data very well. The cumulative distribution function is defined by Eq 1, which is shown below.

=

Eq. 1

From Figure 1 it can be seen that the data is a good candidate to be modeled as an exponential distribution. This is evident by how closely all of the data points lie to the regression line. From this graph the rate value, can be estimated as the slope of the line.

= 0.126



Figure 1: Relationship Between T and Cum. Dist. Function

**Part B)**

**1)**

Here, the maximum likelihood will be used to estimate For this to happen, first the exponential distribution must be defined as it is in Eq. 2.

=

Eq. 2

From Eq. 2 the likelihood function is defined by Eq. 3.

L(,, ,) =

Eq. 3

Because the sequence is made up of independent terms, the likelihood function is equal to the product of their densities.

L(,, ,) = )

=

=

Eq. 4

Now, the log of the likelihood function is taken to make the minimization problem easier.

l(,, ,) = ln(L(,, ,))

= nln() -

Eq. 5

Now, the minimization of Eq. 5 must be taken in order to obtain the estimator. That minimization would take the form of Eq. 6.

= arg

Eq. 6

To find the minimization of the log-likelihood function, the first derivation of the log-likelihood with respect to must be set to zero. This can be seen in Eq. 7.

l(,, ,) = 0

Eq. 7

Solving Eq. 7, we find that the estimator, is just the reciprocal of the sample mean. That can be seen as Eq. 8.

=

Eq. 8

Thus, using the maximum likelihood to estimate would yield:

= = 0.1061

**2)**

Determine the region of support using -1.353. To solve for the support region using the value supplied above, Eq. 9 was solved.

Log( = log() = -1.353

Eq. 9

Solving Eq. 9 for the two will provide the support range that is shown below.

0.071 0.146

**3)**

The purpose of this section of the homework is to approximate the 90% confidence interval graphically. The approximation plot can be seen below as Figure 2. The values found were found graphically can be seen below.

0.073 0.151

These values are extremely close to the values that were found previously using the maximum likelihood method.



Figure 2: Quadratic Approximation